Subgame Perfection
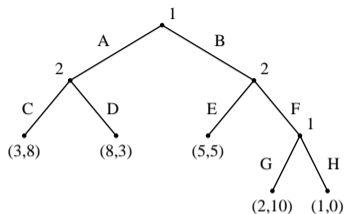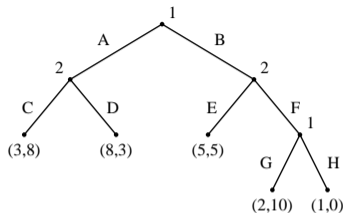
Game Theory Course:
Jackson, Leyton-Brown & Shoham

- There's something intuitively wrong with the equilibrium $(B,H),(C,E)$
  - Why would player 1 ever choose to play H if he got to the second choice node?
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- There's something intuitively wrong with the equilibrium $(B, H), (C, E)$
  - Why would player 1 ever choose to play H if he got to the second choice node?
    - After all, G dominates H for him
  - He does it to threaten player 2, to prevent him from choosing F, and so gets 5
    - However, this seems like a non-credible threat
    - If player 1 reached his second decision node, would he really follow through and play H?

Formal Definition

### Definition (subgame of G rooted at h)

The subgame of G rooted at h is the restriction of G to the descendents of h.

Definition (subgames of G)

The set of subgames of G is defined by the subgames of G rooted at each of the nodes in G.
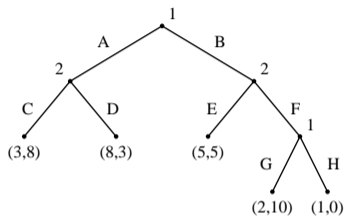
- s is a subgame perfect equilibrium of G iff for any subgame G' of G, the restriction of s to G' is a Nash equilibrium of G'
- Notes:
  - since G is its own subgame, every SPE is a NE.
  - this definition rules out "non-credible threats"
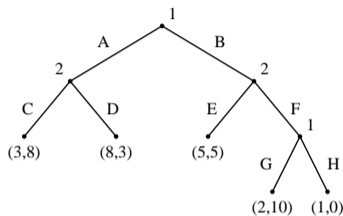
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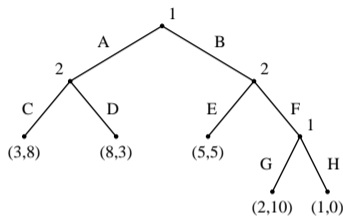
- Which equilibria from the example are subgame perfect?
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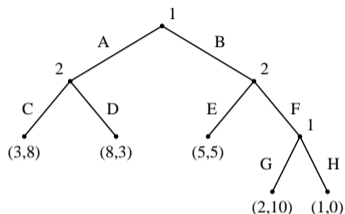
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  - $(A, G), (C, F)$: is subgame perfect
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  - $(A, H), (C, F)$:

Subgame Perfection

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  - $(B, H), (C, E)$: (B, H) is non-credible; not subgame perfect
  - $(A, H), (C, F)$: (A, H) is non-credible, though H is "off-path"